WHIRLING OF SHAFT AND LATERAL VIBRATION ANALYSIS

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ABSTRACT
In mechanical engineering, some machine components can behave differently due to the design of machine elements, manufacturing processes, and selection of materials. To understand basic phenomena of any general dynamic stresses, it is good to understand adequate modeling of the system. To start with, a lateral vibration analysis of the shaft is considered. It is well known that lateral bending, whirling and transverse vibration of propulsion systems phenomenon is not as dangerous as the torsional vibration. This study is focusing on the lateral frequencies and the mode shapes of three different materials. The main issue of the lateral frequency is that could cause resonance and fatigue in the material. In this study, mild steel is used as the benchmark material in this study. While the other two types of materials are aluminum oxide and stainless steel. There will be lumped and distributed model will be considered in this study. The parameters to be determined include the mode shapes and natural frequencies of the whirling shaft. As a conclusion, aluminum oxide is the best material used for this study because it has a high value of natural frequencies to avoid the resonance happened.

Keywords:
Lateral vibration; natural frequencies, mode shapes, resonance, steel, aluminum oxide.

INTRODUCTION

In mechanical engineering, some machine components can behave differently due to the design of machine elements, manufacturing processes, and selection of materials. To understand basic phenomena of any general dynamic stresses, it is good to understand adequate modelling of the system. It is well known that lateral bending, whirling and transverse vibration of propulsion systems phenomenon is not as dangerous as the torsional vibration. A shaft is a rotating machine element, usually circular in cross section, which is used to transmit power from one part to another, or from a machine which produces power to a machine which absorbs power. In general, a lumped-parameter approach is appropriate when the physical object has dimensions that are small relative to the wavelength of vibration. A distributed system is one in which all dependent variables are functions of time and one or more spatial variables [1].

Most shafts are subjected to fluctuating loads of combined torsion and bending with various degrees of stress concentration. For such shafts the problem is fundamentally fatigue loading. Failures of such elements and structures have engaged engineers and researchers extensively in an attempt to find their main causes and thereby offer methods to prevent such failures. Whirling is usually associated with fast speed of the rotating shafts. The speed in the shaft is called as “critical speed” and the act of vibrations called “whirling”. If the speed of the shaft remains to be the same then that will lead to damage in the shaft and will cause a failure. Moreover, if the shaft speed keeps increasing before any other effects happen then the shaft will continue working safely until other effect could interrupt the shaft. Leave one blank line between the heading and the first line of text [1].

Additionally, whirling of shafts occurs due to rotational imbalance of a shaft, even in the absence of external loads, which causes resonance to occur at certain speeds, known as critical speeds. Also, the whirling is happening when the resonance is occurring too. While the shaft is rotating, it could be affected by two forces: the radial and centrifugal forces; which will lead the shaft to move from its “safe” position. Also, the whirling of shafts is a serious problem in the most machines that use long shafts [1].

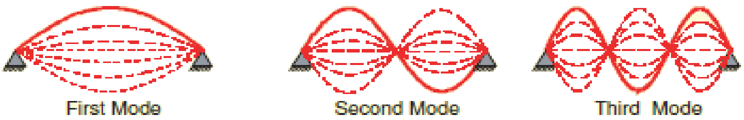

Figure 1: First three modes, Practical review of rotating machinery [1]

There are different modes and shapes formed due to the high speed and the lateral frequency met. It's all controlled by the boundary conditions that could happen. Whenever the speed increase then the frequency and the mode shapes will form more as it's mentioned in figure 1 these are the first three mode shapes in the simply supported boundary condition.

Based on the Vibration of Rotating Shafts 1959 journal by R.E. D. Bishop, the vibration of rotating shaft was established on rotational basis by Jeffcott. He noticed the effect that happen on the shaft from bending in the same or other as he follows the principle modes that occur when it reaches the running speed with natural frequency and explain the configuration of the shaft during the vibration. For the mathematical Jeffcott focused on the unbalance mass on the thin shaft then he explains the whirling of shaft in fundamental mode.

On the other hand, Lewis did the analytic process of the running shaft through the critical speed. While the other scientists Golomb and Roseberg have studied the whirling of shaft as a problem on uniform shaft which can transmit a torque. Also Lewis and Downham did their experimentation based on Jeffcott approach. Apart from that Johnsen who refers specifically to the rotating uniform shaft that is pined at the end from both sides. Tondl examine a different problem most of it is using Jeffcott approach and he assumed that the pin-pin shaft is so far as its unsymmetrical rotates. As William T. Thomson, Professor Emeritus mentioned in Theory of Vibration with Applications Fourth Edition 1993, Rotating shafts at some certain speeds tend to whirl in some complicated ways [2].

A review on materials

The selection of the material is one of the most important elements in engineering design. Also it's an important stage to start the project because some material has its own optimum properties or sometimes it's a combination of properties between two materials to get the best properties needed. The benefits of material consideration are to reduce the cost and also to improve the performance. Additionally, elements of this materials selection process involve deciding on the problem and from those criteria can see which material is the best to maximize the performance. The component that been chosen to discuss on is a solid rod shaft that is used to transmit the movement. There are different materials can be used depend on the cost of the material, the weight and the strength [3].

A. *Mild steel*

The mild steel is one of the good materials that used in the shaft. In this journal there was a study on the elastic shaft that is whirling with the first three mode shapes by three different boundaries of conditions. The boundaries of conditions are simply supported, cantilever and fix end as it's mentioned. All the elastic shafts are suffering from natural frequency and that is depending mostly on the how many degree of freedom they are described. The shaft in the end will suffer from resonance when it matches with the natural frequency. As well as the amplitude is going to be high value. For example, in rotating shaft the resonance is unwanted due to the rotor vibrate strongly and lead to break it [4].

B. Aluminium Oxide and Stainless steel

Each material has different property from young modulus and the yield strength which is important to decide in the software to get it started. Moreover, the author chose the carbon steel as it mentions in the journal. Additionally, the table below shows the other property of the materials.

Table 1: Materials property of shaft [5]

Material	AISI 1018 mild Steel	Aluminum Oxide (Al_2O_3)	Stainless steel (304)
Density p	$7830\,kg/m^3$	$3960\,kg/m^3$	$7800\,kg/m^3$
Young modules E	$205Gpa$	$390Gpa$	$210Gpa$
Poisson Ratio	0.33	0.26	0.3

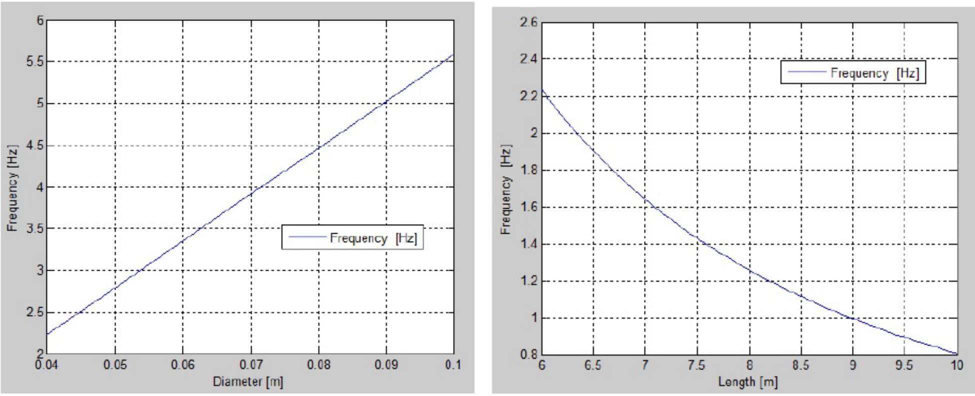

Figure 2: The diameter and length vs the frequency, Numerical and Analytical Analysis of Elastic Rotor Natural Frequency [5]

Figure 2 shows that the frequency is increasing when the diameter is getting bigger these values is being presented by the theoretical calculation. On the other hand, the frequency is decreasing when the length is increased.

Table 2: The results of the theoretical and simulation, Numerical and Analytical Analysis of Elastic Rotor Natural Frequency [6]

Type of supports	First nat. fre. [Hz]		Second nat. fre. [Hz]		Third nat. fre.[Hz]	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
Simply sup.	2.23	2.24	8.93	8.87	20.09	19.75
Canti.	0.55	0.79	5.02	4.95	13.95	13.95

<u>*Equations of the manual calculation*</u>

There are two main equations that been used in this study. A distributed parameter system as it's the opposite of lumped parameter system which is could also define as infinite dimensional due to the state space. Also the best example that can mention about this system is partial differential equations. To determine the equation for the vibration of the beam should be conceded that the forces and the moments acting on the element of the beam as it's shown below in the equation (1).

$$w_n = {\beta_n}^2 \sqrt{\frac{EI}{p}} = (\beta_n l)^2 \sqrt{\frac{EI}{pl^4}} \quad (1)$$

As for this equation the represent the boundary condition of this case if it's a simply supported or cantilever which it can get the value of by referring to the Table 3 as shown the first three modes.

Table 3: Euler equation table values, Theory of Vibration [7]

Beam Configuration	$(\beta_1 l)^2$ *Fundamental*	$(\beta_2 l)^2$ *Second Mode*	$(\beta_3 l)^2$ *Third Mode*
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free-free	22.4	61.7	121.0
Clamped-clamped	22.4	61.7	121.0
Clamped-hinged	15.4	50.0	104.0
Hinged-free	0	15.4	50.0

On the other hand, the other system is lumped parameter system. In this system is it's considering that all the mass in the middle/centre of the shaft not like the distributed system which can be less accurate to assume that. However, nowadays people been using the lumped parameter system because they have the programs to run the finite elements comparing to old days where they used the distributed system [8].

$$\omega_n = \left(\frac{k}{m}\right)^{1/2} \quad (2)$$

METHODOLOGY AND FINITE ELEMENT ANALYSIS (FEA)

The first step in this study is to know the natural frequency that could happen to the shaft that rotate under high speed level. These speeds could match with the frequency that lead to fatigue or other issues to the shaft that can cause resonance. The researchers need to develop a 3D shaft modelling before starting the simulation analysis. Table 4 shows the dimensions and specifications of the shaft.

Table 4: Shaft measurements

Shaft type	Solid shaft
Diameter	0.04m
Length	6m

Meshing used in the Abaqus software are two types. The first one is the triangle shape and the second one is the square shape. Additionally, the writer is mixing between the triangle and the square shape to get the most accurate results. However, the meshing size is important to know about too as it's the distance between one element and another. So if the meshing size is large number then the distance will be bigger and the result won't be an accurate. That's why it's important to make it small to get the high number of elements and also get an accurate result. For this project the author decided to use 5.7mm in meshing size to get the accurate. There are many types of boundary conditions that can be applied. However, the author decides to focus on two types only. The first boundary condition is called simply supported (SS) where the material is being supported from the two sides. Additionally, the other type is cantilever (C) where is being fixed from one side.

Table 5: Boundary conditions of shaft

Condition	Section
Simply supported	
Cantilever	

Each material has different property from young modulus and the yield strength which is important to decide in the software to get it started. Moreover, the researcehrs chose the carbon steel as suggested in a previous study and it is reasonable.

RESULTS AND DISCUSSIONS

Figure 3 to Figure 5 show the simulation analysis results of the mild steel whirling shaft. Based on the results, it shows that the natural frequency of the first mode for the simply supported shaft is 2.19Hz. Figure 4 shows that the natural frequency of the second mode for the simply supported shaft is 8.78Hz and Figure 5 shows that the natural frequency of the second mode for the simply supported shaft is 19.75Hz.

<u>*Mild Steel*</u>

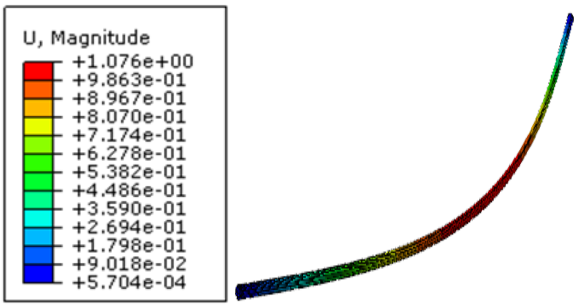

Figure 3: First mode simply supported 2.19 Hz

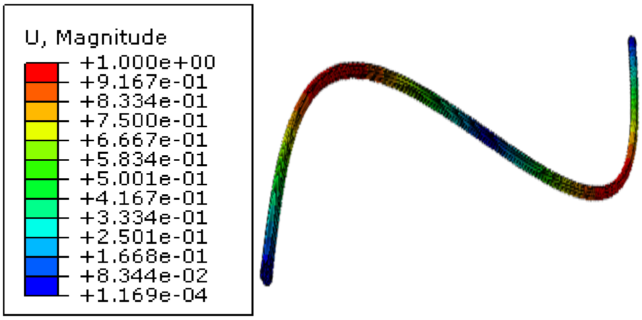

Figure 4: Second mode simply supported 8.78 Hz

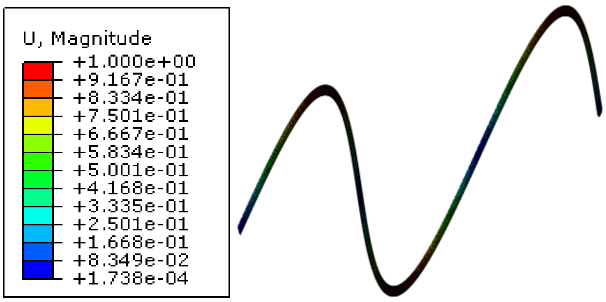

Figure 5: Third mode simply supported 19.75 Hz

These results later are tabulated in Table 6. Table 6 shows the validation of the theoretical calculations with the simulation analysis results with the percentage of errors. From Table 6, Table 7 and Table 8, it shows that Aluminium oxide has the highest natural frequency values.

Table 6: Lumped parameter system results of mild steel

Type and shape	Theoretical (Rad/s)	Theoretical (Hz)	Simulation (Hz)	% error
Mild Steel AISI 1018				
Lumped				
First mode SS	**9.84**	**1.56**	**None**	**None**
Second mode SS	**39.4**	**6.26**	**None**	**None**
Third mode SS	**88.7**	**14.13**	**None**	**None**
First mode C	**2.46**	**0.4**	**None**	**None**
Second mode C	**9.84**	**1.56**	**None**	**None**
Third mode C	**22.16**	**3.52**	**None**	**None**

Table 7: Lumped parameter system results of aluminium oxide

Type and shape	Theoretical (Rad/s)	Theoretical (Hz)	Simulation (Hz)	% error
Aluminum oxide (Al2O3)				
Lumped				
First mode SS	**19.1**	**3.04**	**None**	**None**
Second mode SS	**76.4**	**12.15**	**None**	**None**
Third mode SS	**171.9**	**27.36**	**None**	**None**
First mode C	**4.77**	**0.76**	**None**	**None**
Second mode C	**19.1**	**3.04**	**None**	**None**
Third mode C	**42.99**	**6.84**	**None**	**None**

Table 8: Lumped parameter system results of stainless steel

Type and shape	Theoretical (Rad/s)	Theoretical (Hz)	Simulation (Hz)	% error
Stainless steel (304)				
Lumped				
First mode SS	**9.98**	**1.58**	**None**	**None**
Second mode SS	**39.94**	**6.36**	**None**	**None**
Third mode SS	**89.9**	**14.3**	**None**	**None**
First mode C	**2.49**	**0.39**	**None**	**None**
Second mode C	**9.96**	**1.58**	**None**	**None**
Third mode C	**22.43**	**3.57**	**None**	**None**

Table 9 shows the values from the simulation analysis in simply supported and cantilever aluminium oxide shaft for the first three mode shapes and it being validate with the theoretical calculations to see how much is difference between each results. This table is important because it shows that simulation result is much closer to the publish journal which mean it's accurate.

Table 9: Distributed system results of mild steel

Type and shape	Theoretical (Rad/s)	Theoretical (Hz)	Simulation (Hz)	% error
Mild Steel AISI 1018				
Distributed				
First mode SS	**14.03**	**2.23**	**2.19**	**1.8%**
Second mode SS	**56.15**	**8.93**	**8.78**	**1.7%**
Third mode SS	**126.38**	**20.1**	**19.75**	**1.7%**
First mode C	**5**	**0.79**	**0.77**	**2.6%**
Second mode C	**31.27**	**4.97**	**4.85**	**2.5%**
Third mode C	**87.7**	**13.96**	**13.59**	**2.7%**

Table 10: Distributed system results of aluminium oxide.

Type and shape	Theoretical (Rad/s)	Theoretical (Hz)	Simulation (Hz)	% error
Aluminum oxide (Al2O3)				
Distributed				
First mode SS	**27.2**	**4.33**	**4.26**	**1.6%**
Second mode SS	**108.9**	**17.33**	**17.33**	**0%**
Third mode SS	**174.32**	**39**	**38**	**2.6%**
First mode C	**9.7**	**1.54**	**1.50**	**2.67%**
Second mode C	**60.67**	**9.65**	**9.42**	**2.4%**
Third mode C	**170.14**	**27.1**	**26.4**	**2.65%**

Table 11: Distributed system results of stainless steel.

Type and shape	Theoretical (Rad/s)	Theoretical (Hz)	Simulation (Hz)	% error
Stainless steel (304)				
Distributed				
First mode SS	**14.23**	**2.26**	**2.22**	**1.8%**
Second mode SS	**56.95**	**9.06**	**8.91**	**1.6%**
Third mode SS	**128.17**	**20.4**	**20.04**	**1.8%**
First mode C	**5.07**	**0.81**	**0.79**	**2.5%**
Second mode C	**31.7**	**5.05**	**4.97**	**1.6%**
Third mode C	**89**	**14.16**	**13.97**	**1.4%**

Table 10 shows the results of stainless steel from the theoretical calculations and simulation analysis. Also it shows the percentage of error and the difference between the theoretical and simulation results. From the error percentage it’s noticeable that the value is less than 3% which is consider as accurate for this project.

Table 11 shows the values of stainless steel shaft with the first three mode shapes of the two boundary conditions. The table also shows all the results from calculation and simulation with the error column to observe the difference between theoretical calculation and simulation result.

CONCLUSION

As a conclusion, this project has achieved its aims and objectives successfully. The natural frequencies and the mode shapes of the shaft using theoretical calculation method were obtained. The comparison between theoretical values and the simulation is calculated, from the results it shows the different frequencies of the first three mode shapes. There were three different materials of shafts to study the effect of the material on the lateral frequency. From the results above we can see each case and how it reacts to the frequencies. As the theoretical values have been obtained with the simulation and the percentage of error show the accuracy of these values. Later on the values have been verify with published paper for extra confirmation and it was showing low percentages of errors. From the results, it shows that Aluminium oxide has the highest natural frequency values and this material is the best material to avoid resonance in mechanical machines especially in shaft. The main point of this research is to avoid that type of dangers phenomenon which is known as 'resonance' which will lead to deflection and causes the structures to fail unexpectedly.

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